INCOME RESEARCH + MANAGEMENT

An Options-Based Approach to Quantify CMBS Risk

Historically, securitized tranches have exhibited more downside price volatility and principal loss than equivalently rated corporate bonds. This is largely due to the leverage of securitization technology on the underlying collateral performance. As a complement to the ratings, we propose that investors adopt an option-based approach to quantify expected principal loss under normal and leptokurtic collateral outcomes. We demonstrate the effectiveness of this method by back-testing five-years of CMBS issuance comprised of 447 deals backed by collateral of over $340bn in commercial real estate loans.

Overview

Securitization technology has suffered more than its share of failures over the past few decades. Whether it's the subprime meltdown, aircraft ABS extensions, or extreme CLO price volatility, the embedded leverage in securitizations can exacerbate collateral weakness into outsized principal risk. Although securitized option cost modelling is often thought of in the context of principal *timing* risk, financial options are synonymous with insurance protection which means that this approach may also be used to measure principal *loss* risk.

The search for yield often tempts investors into moving down the securitized capital stack when nominal spreads are wide to equally rated corporate bonds. But in doing so, investors implicitly assume heightened loss potential unique to debt tranching. As a result, they should demand additional nominal spread consistent with the incremental amount of default and severity risk assumed. But exactly how much spread? That is left to the investor to quantify.

For example, let's say an investor has a choice between two bonds – a BBB CLO tranche offered at +200bps/Tsy and a BBB unsecured corporate bond offered at +150bps/Tsy. A recession hits and the risk of default rises – the BBB corporate bond has a 10% chance with an expected recovery of $0.80, while the BBB CLO also has a 10% chance of default but an expected recovery of $0.00. Despite having the same default risk, the BBB corporate now trades at $98 while the BBB CLO trades at $90. Why? Because the market prices each based not on the probability of default but on *total expected loss*.

An option-based framework can quantify this expected loss potential with a credit insurance approach. The concept of **securitized credit option cost** shares similarities to the OAS methodology found in MBS, including its sensitivities to time, volatility, and moneyness.[1] However, we are measuring negative *credit* convexity rather than negative *cashflow timing* convexity, so we use collateral performance volatility instead of interest rate volatility as the basis of outcome uncertainty.

Securitized tranching is designed to provide degrees of principal protection for investors with differing risk/return appetites. But as an investor, it's hard to know your appetite if you are not measuring risk properly. If investors can assess this risk in a systemic and dispassionate way then they can make more informed relative value decisions which should, in turn, maximize their risk-adjusted returns over the long run.

Merton Model Approach

Financial options are synonymous with insurance. In securitized structures, this insurance is the guarantee of 100% principal return for a given tranche provided the tranches junior to it have principal payments remaining. This layer of protection is delineated by a tranche's *attachment* point (collateral % point at which enhancement is gone) and its *detachment* point (collateral % point at which principal is entirely written-off). Beneath all tranches is an equity layer. The equity is sized, at issuance, to absorb the expected rate of collateral loss over the life of the deal and is essentially an insurance deductible for the debt tranches. In the vast majority of securitized issuance, this deductible is not exceeded.

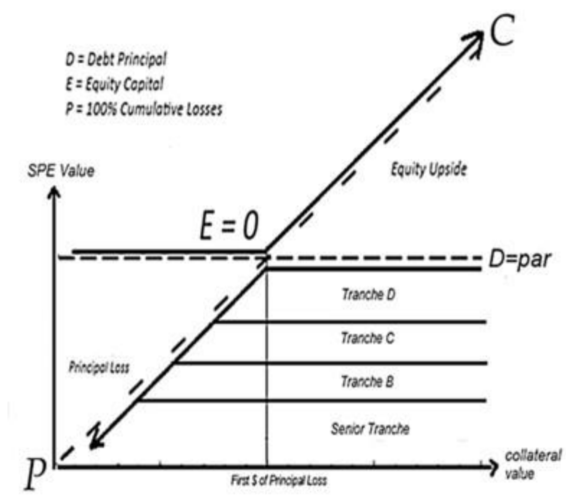


Fig 1: The Merton Model with Debt Tranching

The Merton Model is a theoretical construct in corporate finance which provides a means to conceptualize the debt and equity relationship through options. Equity investors have in-the-money call options on the value of the firm, or, in securitization, the residual value of the collateral. Given put-call parity, the debt holders therefore have written out-of-the-money put options to those equity holders.[2] These puts are exercised if the equity holders choose to default on their debt service obligations in exchange for relinquishing collateral

[1] Quantifying MBS option cost typically involves pricing swaptions to value the risk of a security's cashflow timing materially changing in the future.

[2] Put-Call Parity states that a Protective Put equals a Fiduciary Call.

ownership.[3] Investors in the junior-most tranche then become the de facto owners of the firm's assets. This is in exchange for the requirement to service the securitization's outstanding debt going forward (Fig 2). Lastly, because the debt's legal maturity date is the specific date in the future when principal is repaid, these options are modelled with European expiries.[4]

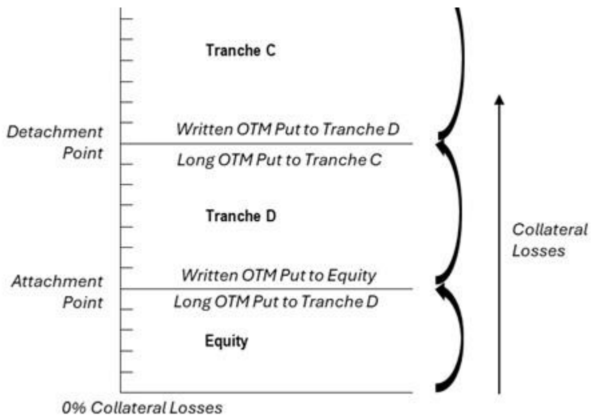


Fig 2: Incremental Collateral Losses transfer SPE "Ownership"

This construct is similar that found in credit default swaps. Equity holders purchase insurance protection on the par value of a firm's debt in exchange for paying recurring coupons. The value of that insurance increases when the maturity extends, the credit volatility rises, and/or the debt-to-equity ratio expands (i.e., the enhancement level in a securitized tranche).

Regarding the last point, the tranche's enhancement is not the only structural aspect which impacts its insurance price. The thickness of the tranche, or its relative size in the deal, is also directly tied to the principal risk. We show this in Fig 3 by dividing the tranche into incremental slices of collateral (△), effectively making each slice its own tranche of principal.

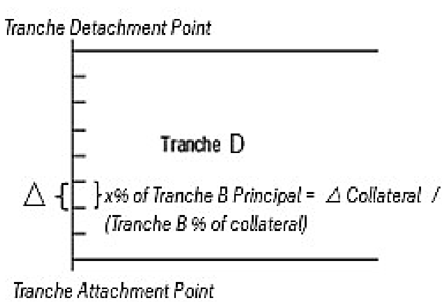


Fig 3: Tranche Principal Leverage on an Increment of Collateral

Written Bear Put Construct

Insurance on an incremental collateral slice may be synthetically replicated with a **Written Bear Put Spread** (Fig 4). This then maps to a specific portion of principal exposure.

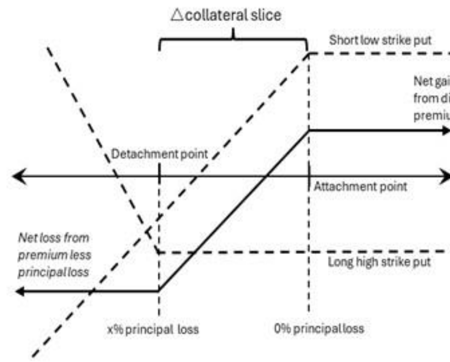


Fig 4: Written Bear Put Spread with Incremental △ Collateral Exposure

A Written Bear Put Spread generates income from the difference between the high and low strike premiums. If we apply the *Fundamental Theorem of Calculus* in which *b* and *a* are the tranche attachment and detachment points and *P(b), P(a)* are the corresponding put prices, then we arrive at:

$$P(b) - P(a) = \int_a^b p(\Delta)\, dx$$

If $p(\Delta)$ is the put price difference for incremental collateral slices from 0%-100%, then we may quantify the total insurance cost via the Black-Sholes partial differential equation:

$$\frac{dV}{dt} + \frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + r\left(S\frac{dV}{dS} - V\right) = 0$$

Assuming that *r=0%* (for simplicity) and $p(\Delta)$ is V:

$$\frac{dp(\Delta)}{dt} = \Delta Q = -\frac{1}{2}\sigma^2 S^2 \frac{d^2 p(\Delta)}{dS^2} = -\frac{1}{2}\sigma^2 S^2 \Delta\Gamma$$

The option's time value, Theta (Θ), is therefore a proxy for nominal spread erosion while gamma (Γ) is for credit convexity risk. The cost to insure each Δ collateral slice becomes the rate of change in the prices of the puts covering the collateral pool.[5] The **Credit Option Cost (COC)** then becomes a sum of these put price differentials from the tranche's attachment to detachment points. Lastly, subtracting the summed credit option cost from the tranche's nominal spread is its **C-OAS**:

Credit Option Adjusted Spread (C-OAS) = Nominal Spread - COC

C-OAS is therefore our best quantitative estimate for the average loss-adjusted income we'd earn on a tranche with an infinite set of probabilistically weighted randomly distributed set of outcomes in collateral valuation at maturity.

[3] In corporate finance, this is filing for bankruptcy. In securitization, this is when collateral write-downs wipe out the principal balance of the tranche.
[4] European expiry given the holder the option to excise on one specific date.

[5] In extremis, binary collateral outcomes, which is essentially an infinitely small tranche thickness, have the highest credit convexity and the most option cost.

Kurtosis Risk in Commercial Real Estate

The Black-Scholes Model (BSM) is perhaps the most popular option model in finance, but it has limitations in its applicability to many types of securitized collateral. BSM's roots are in periodic bilateral asset price returns (i.e., common stocks), geometrically linked to produce lognormally distributed terminal asset valuations under a constant volatility assumption.[6] Nonetheless, we use BSM to model the valuation distributions of assets such as commercial real estate (CRE).

CRE valuation is notoriously idiosyncratic over time. Thus, we may choose to adjust the range of collateral valuation outcomes from normally distributed to leptokurtotic or "fat-tailed". We do this with the inclusion of a variance neutral stochastic element incorporated directly into our distribution's volatility parameter via **Jensen's Inequality:**

$$Jensen's\ Inequality \begin{cases} PDF_{Normal}\left[\frac{1}{2}\sigma\left(\sqrt{(1+a)}+\sqrt{1-a}\right)\right] \quad - \\ \frac{1}{2}\left[PDF_{Normal}\left(\sigma\sqrt{(1+a)}\right)+PDF_{Normal}\left(\sigma\sqrt{(1-a)}\right)\right] \\ where\ 0 < a < 1 \end{cases}$$

Applying Jenson's Inequality to a normal distribution transforms it into a leptokurtotic distribution. For example, if we wish to add +2 units of kurtosis then we set the stochastic constant (a) to 0.817. Since asset volatility scales with time ($S_0\sigma_a\sqrt{t}$) lengthening the maturity increases the leptokurtosis (Fig 5). This leads to proportionally higher credit option costs further up the securitized stack.

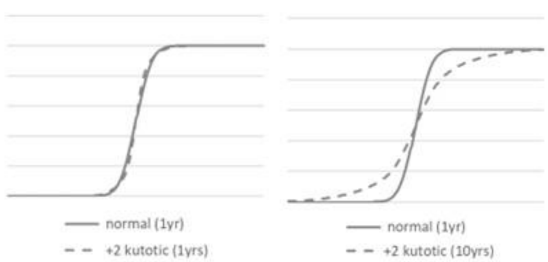


Fig 5: Normal vs. Kurtotic Distribution for 1yr and 10yr Maturities

A Case Study in Single Asset CMBS

CMBS is an example of securitization undergoing a rapid evolution in both structure and collateral. The more traditional Conduit CMBS remains well diversified in its geography, usage, and tenants. Alternatively, the SASB sub-sector which includes Single-Asset (SA) and Single-Borrower (SB) issuance, typically incorporates a single property or type of properties. We apply the traditional Black-Scholes option model to the Conduit CMBS tranches but add excess kurtosis to the SASB issuance, which has historically been more idiosyncratic in its valuations.

One prominent example of this idiosyncratic behavior is in collateral of the BWAY 2015-1740 securitization (an office tower located at 1740 Broadway in Manhattan). The deal was issued in 2015 as a 10yr securitization of the property's commercial mortgage and had a senior tranche (A) and five subordinate tranches (B-F). At the time, the building was appraised for $605mm, which implied an LTV of 51%. The deal had $300mm of investment grade (IG) rated tranches (Fig 6):

Tranche	Size ($1000's)	Original S&P Rating	% of Deal	Enhancement w/ LTV	Weighted Average Life (yrs)	Spread @ Issuance (bps)
A	157,000	AAA	26.0%	74.0%	9.91	150
B	38,620	AA-	6.4%	67.7%	9.91	175
C	26,743	A-	4.4%	63.2%	9.91	195
D	32,806	BBB-	5.4%	57.8%	9.91	245
E	44,573	BB-	7.4%	50.5%	9.91	340
F	7,758	B+	1.3%	49.2%	9.91	650
Equity	297,500	NR	49.2%	--	--	--

Fig 6: BWAY 2015-1740 Capital Structure and Pricing at Issuance

We apply conservative option-model estimation parameters of 0% mean appreciation, 15% implied volatility, and a 10-year expiry for each tranche.[7] We run option costs under two scenarios – normal (#1) and +2 units of additional kurtosis (#2). We then subtract these two option costs from the nominal spread of each tranche. We use these C-OAS results in a relative value context to ascertain which tranche offers the highest risk-adjusted return potential to maturity (Fig 7):

	Scenario #1			Scenario #2		
Tranche	Nominal Spread @ Issuance (bps)	Less Credit Option Cost with Normal Kurtosis (bps)	Equals C-OAS with Normal Kurtosis (bps)	Nominal Spread @ Issuance (bps)	Less Credit Option Cost with +2 Excess Kurtosis (bps)	Equals C-OAS with +2 Excess Kurtosis (bps)
A	150	0	150	150	7	143
B	175	7	168	175	56	119
C	195	18	177	195	94	101
D	245	34	211	245	137	108
E	340	66	274	340	200	140

Fig 7: Credit Option Cost and C-OAS with Normal and Excess Kurtosis

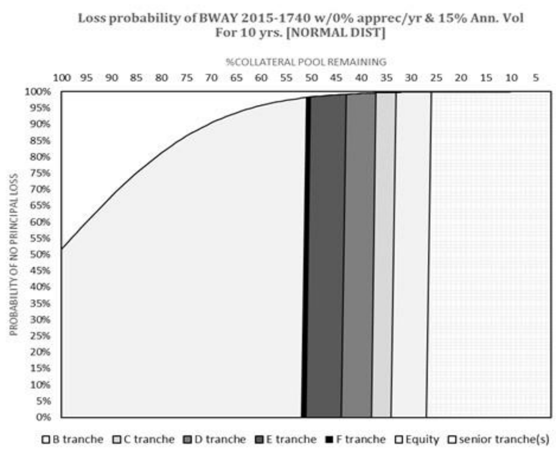
These results show the difference excess kurtosis can make in our relative value assessment between the senior and senior-most subordinate tranche. Since extreme appraisal outcomes cannot be ruled out in SASB CRE, the additional kurtosis added in #2 was key to haircutting the projected risk-adjusted spreads to levels which weren't compensatory for the risky nature of a single office property in midtown Manhattan.

C-OAS quantifies the risk relative to the incremental nominal spread pick-up when an investor moves down the capital

[6] The less elegant but more flexible binomial lattice modeling fits most other securitized collateral types, but we will save that for another paper.

[7] CRE Cap Rates were close to all-time lows in 2015.

stack. This approach can also facilitate relative value assessments across deals and collateral types.



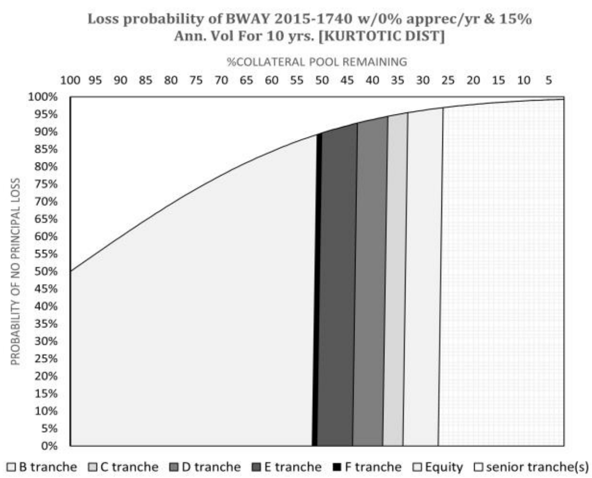


Fig 8: Non-Zero Probability of Loss Expands w/Fat Tail Distributions

As seen, excess kurtosis provisions for the remote but ever-present possibility of catastrophic decline in collateral value. This was indeed the case for BWAY 2015-1740. The office property was liquidated in 2024 for $117mm in net proceeds, an 80+% decline which effectively wiped out the principal of all the subordinate tranches and costing the seniors $0.24 of principal loss. Naturally, C-OAS did not predict this dire of an outcome, but it did tilt the relative value towards the seniors in scenario #2. While this is an extreme example, it demonstrates that C-OAS may highlight instances where the investor is be undercompensated for the structural risk posed by the enhancement levels, thicknesses, and maturity length of the deal.

5-Year CMBS Back-Test

CRE valuations have suffered extreme corrections over time. This makes it an ideal testing ground for assessing the efficacy of C-OAS. To this end, we calculate C-OAS in a variety of manners. First, we define our sample set of deals by running the following filters on the entirety of US dollar denominated CMBS deals issued between 2015-2020:

- Listed on Bloomberg as having type *Conduit* or *SASB* (767 deals).
- Contains at least one senior and one subordinate tranche with:
  - Principal Outstanding as of 12/31/2024 (508 deals).
  - An IG rating at issue by one or more NRSRO's (459 deals).[8]
  - Pricing available on 12/31/2024 by Bloomberg (447 deals).

This leaves us with 894 qualifying senior and senior-most subordinate tranche candidates to potential buy in our CMBS-only portfolio. To assess this, we run C-OAS with the following estimation parameters:

- 0% mean/expected collateral appreciation.
- 15% annualized collateral volatility.
- Time-to-maturity set as the legal final (not 'expected' final).[9]
- Enhancement equal to total LTV on original collateral appraisal.

Applying these constraints, we calculate a C-OAS for each of the two tranches in each deal to choose the one with the highest risk-adjusted return potential. We do this for each of the following four alternative portfolios:

1. No excess kurtosis added.
2. +2 units of excess kurtosis added to all CMBS deals.
3. +2 units of excess kurtosis added to only SASB CMBS.
4. +1 unit of excess kurtosis added to only SASB CMBS.

In each portfolio, we calculate the C-OAS difference between the senior and subordinate tranches at the time of purchase.[10] We then automatically buy and hold one unit of the tranche with the highest C-OAS. We assign spread-based return estimates for each portfolio based on:

- Senior tranches have a spread of zero.
- Subordinate tranches have a return equal to their incremental starting spread over that of their senior tranche.[11]

The realized risk is the price difference between the senior and subordinate tranches from the time of purchase to 12/31/2024. This risk serves as a proxy for the market's change in the perceived principal risk differential between the senior and senior-most subordinate tranches from the time of issuance to 12/31/2024.[12]

[8] This includes S&P, Moody's, Fitch, DBRS, and KBRA.
[9] An open-ended extension option is held by CRE servicers (typically exercised in 1yr. increments) in the event that the balloon payment cannot be made at expected maturity.
[10] Defined as earlier of issue pricing date or the earliest pricing date available.
[11] Book Yield is defined as the Yield on the tranche at the time of purchase.
[12] Markets in late 2024 were highly liquid, so we assume that the driver of this price difference is more fundamental than technical.

Results & Conclusions

Table 1 summarizes the results of our CMBS back-test. The two control portfolios – Portfolio 6, which buys only senior tranches (minimum risk) and Portfolio 1, which buys only subordinate tranches (maximum return) – define our outcomes *in extremis*. The midpoint between the two represents the benchmark, which holds all purchasable tranches in equal weights (i.e., 50% seniors/50% subordinates). Portfolios 3, 4, and 5 are the actively selected portfolios which incorporate excess kurtosis into the C-OAS relative value decision. The active bottom-up alternative to the benchmark (4a) is a variation on portfolio 4 in that it weights seniors and subordinates equally overall (50%/50%) but buys only one tranche per deal. In all cases, the portfolios with added kurtosis outperformed their counterparts (Fig 9):

	Total Subordinated Tranches in Portfolio	Portfolio Level Price Erosion from Sub. Tranches	Number of Sub Tranches Downgraded Below IG	Number of Holdings Priced < $50 on 12/31/24	Portfolio Level Book Yield Pickup from Sub Tranches
1. Buy Subs (Only)	447	-4.93	19	10	55
2. Buy Subs C-OAS > Senior C-OAS	409	-4.58	18	9	53
3. Buy Subs C-OAS (SASB Kurtosis = +1) > Senior C-OAS	311	-3.08	6	2	46
4. Buy Subs C-OAS (SASB Kurtosis = +2) > Senior C-OAS	301	-2.74	2	1	45
4a. Equal Weighted Seniors and Subordinates	301	-2.04	2	1	34
5. Buy Subs C-OAS (All Kurtosis = +2) > Senior C-OAS	62	-0.52	2	1	13
6. Buy Seniors (Only)	0	0	0	0	0

Table 1: Results of CMBS Back-Test

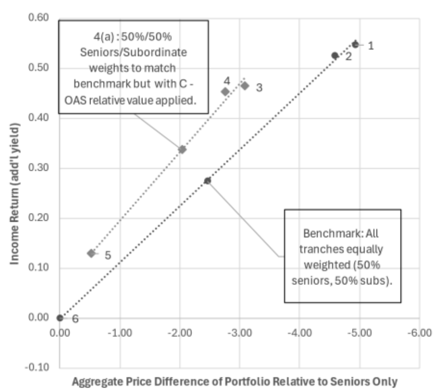


Fig 9: The use of C-OAS improves Risk-Adjusted Returns.

These results also demonstrate that the C-OAS approach did not select the bulk of subordinate tranches which suffered adverse outcomes, including severe downgrades to below IG and/or extreme price downside. The question is, why?

This back-test was specifically designed to measure the relative structural risk for a given tranche and apply it as a haircut to the relative return it offered. It is possible that, if a tranche offers enough compensation, then its fragility will not deter it from being added to the portfolio. However, in hindsight, the market seemed to overlook the potential for principal loss presented by this structural fragility and instead priced the SASB subordinates with credit option costs consistently higher than their marginal pick-up in spread.

Fig 10 shows the relative option cost verses the nominal spread for all subordinate tranches in Portfolio 4. Subordinate tranches with adverse outcomes (price < $50 and/or downgraded below IG) are outlined in black. These are almost entirely SASB deals. One explanation for this is that the scarce additional spread that these SASB subordinate tranches offered was offset, in some cases, by the tranches pricing at discounts. In these cases, if the tranche pays par at the anticipated repayment date (and that date is well before legal maturity), then investor accrues price upside. Regardless, we believe a risk-adverse investor should use the more conservative spread to legal maturity for risk assessment.

Conduit CMBS fared far better, and, in many cases, over-compensated investors for subordinate tranche risk. Even if C-OAS users added +2 units of kurtosis to Conduit (portfolio 5), 51 subordinate tranches out of 293 conduit deals (~17%) would have been bought while only 11 subordinate tranches out of 159 (~7%) SASB deals would have been purchased. This may have been more risk-adverse than necessary, but it still produced better risk-adjusted returns than portfolio 6.

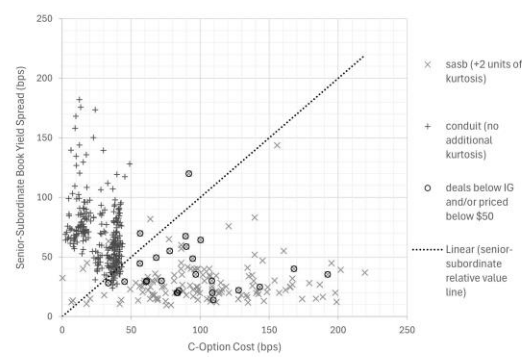


Fig 10: SASB deals had high option costs and low YTM spreads at issuance.

These results are pronounced largely because CRE valuations have undergone an especially difficult period of correction. However, given the asymmetric risk to principal, bond investors should take heed of this historical experience (and others). It, once again, proves that in securitized investing, an error of omission is far kinder on the investor's long-term track record than an error of inclusion.

Finally, this approach is not a substitute for fundamental analysis of the collateral itself. Also, a more sophisticated enterprise risk model, such as one grounded in Monte Carlo might be preferred. However, C-OAS is extremely flexible, scalable, and explainable amidst a hectic trading desk where analysts, traders, and portfolio managers must engage on relative value decisions throughout the busy trading day.

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